FOWLER'S
LONG-SCALE CALCULATOR

VEST POCKET MODEL (RX)

For Engineers, Draughtsmen and Science Students

(Supplied Complete in Leather Case)

Fowler's Calculators, like all Slide Rules, are based on the properties of logarithms, which enable Multiplication to be effected by addition, and Division by subtraction. These operations are made mechanical by moving circular scales with logarithmic distances marked on them, which can be added or subtracted from one another.

The instruments are all of standard size and price, and consist of a circular nickel case fitted at front and back with glass cover. Within the case are two rotating metal dials $2\frac{3}{8}$ ins. diameter (see Figs. 1 and 2, page 3) on which the engraved scales are printed. These scales are operated by thumb nuts outside the case.

On each cover glass there is a fixed radial datum line. The front dial is also provided with a rotating radial line or Cursor.

No Cursor is fitted to the back dial.

The instrument is contained in a separate velvet-lined nickel protecting case.

Every detail is made in our own factory by the aid of special machinery to secure the greatest accuracy and excellence of workmanship.

Features of Fowler's Calculators.

Dials Synchronised.—The instruments are so constructed that when the zero line of front dial is set under Front Datum line, the Front Cursor line and Back Datum line always give similar readings on the outer scale. This is sometimes convenient for directly transferring values of functions from Back dial to Front dial, and is specially handy for determining Cube Roots.

Accuracy of Results.—With the "Short-scale" of the front dial calculations of all kinds can be made rapidly and accurately to two and frequently three significant figures. If greater accuracy is required, the "Long-scale" can be used, which will give results accurate to three and often four significant figures.

Advantages over Straight Slide Rules Scales always clean. No slides to stick or become slack. No switching of slide; quicker to set. Results more accurate. Instrument goes in waistcoat pocket, and not affected by temperature, dirt or moisture.

The makers recommend the "Long-scale" type of instrument (here illustrated) to Engineers, Draughtsmen and Science Students for all-round purposes as the most comprehensive and accurate pocket calculator ever made. Other special types are more convenient for special purposes, and descriptive circulars of these will be sent free on application.

DESCRIPTION OF DIALS
of Fowler's " Long-scale " Calculator.

The Front Dial, Fig. 1, comprises an outer " Short-scale " complete in a single circle 6·7" in circumference, which can be used for Multiplication and Division in the same way as in other types of Fowler's Calculators.

The six inner circles form a similar continuous " Long-scale " 30" in total circumference. It is used just like the single circle or " Short-scale," though the " Short-scale," is sometimes convenient for finding an approximate result to determine the precise circle of the " Long-scale " on which to read the very accurate result.

Whether to use the " Short-scale " or the " Long-scale " is entirely at the choice of the operator according to his convenience and the degree of accuracy required.

The Back Dial, Fig. 2, comprises seven scales, reckoning inwards as follows :—

No. 1.—A scale similar to the multiplying and dividing scale of the front dial.

No. 2.—A scale of Reciprocals exactly similar to Scale No. 1, but graduated contra clockwise instead of clockwise.

No. 3.—A scale of Logarithms graduated at intervals of ·01.

No. 4.—A scale of Square roots from 0 to 100. (The values of the squares are read on Scale No. 1.) This scale extends continuously twice round the circumference.

No. 5.—A scale of Log. Sine Values from 5° 45′ to 90°.

From 5° 45′ to 8° graduations every 5′.
    8° to 12° graduations every 10′.
    12° to 30° graduations every 20′.
    30° to 60° graduations every 1°.
    60° to 80° graduations every 2°.

No. 6.—A scale of Log. Tangent Values from 5° 45′ to 45°.

From 5° 45′ to 8° graduations every 5′.
    8° to 12° graduations every 10′.
    12° to 30° graduations every 20′.
    30° to 45° graduations every 30′.

No. 7.—A scale of Log. Sines from 35′ to 5° 45′

From 35′ to 1° graduations every 1′.
    1° to 3° graduations every 2′.
    3° to 5° 45′ graduations every 5′.

Note.—The values of all the functions given on Scales No. 3 to No. 7 can be read directly on Scale No. 1 by means of the Datum line, or their inverse values by reading on Scale No. 2.

Gauge Points.—For facility in setting, both the " Short-scale " and the " Long-scale " of the front dial as well as outer scale of the back dial have the following values specially indicated :—$\sqrt{2}$; $\sqrt{3}$; log$e$ 10 ; π ; gE (gravity English) ; E.H.P. (electrical horse-power) ; $\pi/4$; gF (gravity French).

Operating Nuts.—The thumb nut X, Fig. 1, operates the front dial only.

The thumb nut Y, Fig. 1, operates the Cursor E of the front dial and also the back dial.

HINTS ON THE USE OF CALCULATOR.

Use of Scales.—All the Scales of the dials are logarithmic, and as the distance between the figures 1, 2, 3, 4 . . . . 10 gradually diminish, so does the possibility of sub-division and consequent accuracy of reading. Hence the advantage of a " Long-scale " over a " Short-scale."

Readings and settings of sub-divisions depend entirely on the judgment of the observer, and in estimating fractional values the difference between logarithmic distance and actual distance should be noted. Thus for calculation ·25 is more than a quarter and ·5 more than half way.

Any value may be assigned to the figures of the Scales, but proportionate values must then be accorded the sub-divisions. Thus 6 may stand also for ·6, ·06, ·006, 60, 600, etc., but if taken, say, to represent 60, the sub-divisions would then represent 61, 62, etc., and similarly for any other value accorded to the prime number.

Selection of Proper Units of Measurement.—In making calculations by whatever method or instrument, care should be taken to express the quantities in proper units.

If an area is desired in square feet as a product of linear dimensions, these should be expressed in feet. If desired in square yards the dimensions should be in yards.

It does occasionally happen that some dimensions are in one unit and others in a different unit. For example, in formulæ for beams, breadth and depth are usually expressed in inches, and span in feet. Again, the weights of bars are generally given per foot run, although the sectional dimensions are in inches. Mistakes occur through overlooking these points.

Simplifying Decimals.—In simplifying decimal values regard should be paid to the value of any terminal figures that may be struck off.

For example. If we wish to contract a value such as 15·647, then 15·65 is a nearer approximation than 15·64, because the figure 7 is nearer 10 than 1. If the last figure had been 2 instead of 7, then 15·64 would have been more approximate than 15·65.

If a repeating decimal is less than 5, write it twice and increase the last figure by one. Thus 6·22 becomes 6·23.

If a repeating decimal is greater than 5, it need only be given once, but increased one in value. Thus 6·77 may be written as 6·8.

A misconception of the fractional value of decimals sometimes causes mistakes, especially if there are cyphers to the right of the decimal point.

To avoid confusion, remember that when expressed as a fraction the number of cyphers in the denominator is the same as the number of figures after the decimal point.

For example—

$$3·04 = 3\frac{4}{100} = \frac{304}{100} \qquad ·96 = \frac{96}{100} \qquad ·002 = \frac{2}{1000}$$

Fixing Decimal Point. Mental Calculation.—When possible, put all calculations in a simple fractional form.

The correct position of the decimal place in the answer can be best determined by rapid mental multiplication and division. There are rules for doing this, but they are more trouble to remember than they are worth, and liable to cause mistakes. It is much better for the operator to depend on first principles and a little mental arithmetic.

For example, suppose value of following required :—

$$\frac{6·92 \times 746 \times 19·2 \times 9}{2876 \times 92·5}$$

we could reason mentally as follows :—6·9 is practically 7, and 7 into 2,876 is roughly 400 ; 400 into 746 is roughly 2 ; 2 into 92·5 is roughly 45. This would be in the denominator, and for the numerator we should still have left 19·2 × 9, roughly 170. This divided by 45 would obviously give a value less than 10. In putting down the answer, therefore, we should write all figures after the first one to the right of the decimal point. A rough estimate like this occupies less time to make than to describe, and is safer than any cut and dried rule.

ARITHMETICAL EXAMPLES.

Multiplication.—Front dial. Either the " Short-scale " or the " Long-scale " may be used, but for purpose of illustrating the principle, the description will be first confined to the single outer circle or " Short-scale " No. 1.

Suppose the factors are $a \times b \times c$, etc.
Set a under Datum F by nut X.
Set Cursor E at 1·0 by nut Y.
Set dial by nut X till b is under Cursor E.
Read $a \times b$ under Datum F.
If there is a third factor c :
Set Cursor again at 1·0 by nut Y.
Set dial by nut X till c is under Cursor E.
Read $a \times b \times c$ under Datum F, and so on for any number of factors.

In using the " Long-scale " formed by the six inner circles of the front dial, the setting is precisely the same as described above for the " Short-scale," but the factors a, b, c, etc., are, of course, read exclusively from the " Long-scale."

In reading the answer, the first figure of the result, and hence, the particular circle on which to look for it, may be determined either by a preliminary working on the " Short-scale," or by rapid mental estimation as indicated under " Hints on the Use of Calculator." The value of the first figure of the answer, and whether it represents units, hundreds, thousands, etc., determines the position of the decimal point and the reading of the following figures of the answer from the sub-divisions of the scale.

Division : Fractions.—If division of simple form $\frac{a}{m}$ use Scale No. 1.

Set a under Datum F by nut X.
Set Cursor E to m by nut Y.
Set dial by nut X till 1·0 is under Cursor.

Read $\frac{a}{m}$ under Datum F.

If division of form $\frac{a \times b}{m}$ proceed thus :—

Set a under Datum F by nut X.
Set Cursor E to m by nut Y.
Set dial by nut X till b is under Cursor E.
Read answer under Datum F.

The numerator of the fraction must contain one more factor than the denominator. If there are not enough multipliers or divisors in the fraction, insert the figure 1 as often as required, e.g. :—

$$\text{operate } \frac{a \times b \times c}{m} \text{ as } \frac{a \times b \times c}{m \times 1}$$

$$\text{operate } \frac{a \times b}{m \times n} \text{ as } \frac{a \times b \times 1}{m \times n}$$

The position of the decimal point in a calculation, if one is required, is found as explained above, either by inspection or rapid mental calculation.

Note.—The Datum F is only used for first multiplier and answer.

The Dial is turned only for multipliers.
The Cursor E is turned only for divisors.

Instead of operating as described, a complete fraction may, of course, be worked by multiplying all the figures in the numerator and dividing this product by the product of all the figures in the denominator. The choice is a matter for the operator.

Squares and Square Roots.—These are given by a single setting of the back dial and reading directly with the Datum line from Scale No. 1 to Scale No. 4, or vice versa. The numbers on Scale No. 4 are the square roots of those on Scale No. 1.

In fixing magnitude of square of a number, remember that—

Square of any number between 1 and 10 lies between 1 and 100.

Square of any number between 10 and 100 lies between 100 and 10,000.

Square of any number less than unity is less than the number.

Square root of any number less than unity is greater than the number. For example :—

$$\left(\frac{1}{10}\right)^2 = \frac{1}{100} ; \left(\frac{3}{7}\right)^2 = \frac{9}{49} ; \quad \sqrt{\cdot 64} = \cdot 8 \quad \sqrt{\frac{1}{100}} = \frac{1}{10}$$

Rule.—If the number has an odd number of digits, read the square root on the smaller circle of square root scale.

If the number has an even number of digits, read root on the larger circle of square root scale.

Cubes, Cube Roots, Nth Powers, Nth Roots.—If the Calculator happens to be fitted with a Cube Root back dial, instead of the one shown in Fig. 2, the values of cubes and cube roots can be read directly at one setting, as explained in the case of squares and square roots. Remembering that:—

Cube of any number between 1 and 10 lies between 1 and 1,000.

Cube of any number between 10 and 100 lies between 1,000 and 1,000,000.

Cube of any number less than unity is less than the number.

Cube root of any number less than unity is greater than the number.

$$\left(\frac{1}{10}\right)^3 = \frac{1}{1,000} \qquad \sqrt[3]{\frac{1}{27}} = \frac{1}{3}$$

If the Calculator is not fitted with a Cube Root back dial, the cube root value can be obtained like that of any Nth Root (where N may be whole or fractional) by means of logarithms from the following relation :—

If A is a number and $x = A_n$.
Then $x = n \times \log. A.$

The fact that the front and back dials of the "Long-scale" calculator are synchronised, and that the "Long-scale" extends over 6 circles, enables cube-roots to be got by direct reading because

$$\sqrt[3]{x} = \sqrt[6]{x^2}$$

For example, let x be the number whose cube root is required.

First : Set zero-line on front (Long-scale) dial, under Datum line on cover glass.

Second : Set number x on square root scale of back dial under Datum line, then the reading under datum on outer scale of back dial is x^2; so also is the reading on outer scale of front dial under Cursor line, and the sixth root of this number i.e.,

$$\sqrt[3]{x}$$

is on one of the six circles of the Long-scale of front dial.

The precise circle is easily determined by bearing in mind that $10^3 = 1,000$, $20^3 = 8,000$, $30^3 = 27,000$, $40^3 = 64,000$, etc., and therefore that for any number.

Between 1 and 1,000 the cube-root is between 1 and 10
| ,, 1,000 ,, 8,000 ,, ,, ,, 10 ,, 20 |
| ,, 8,000 ,, 27,000 ,, ,, ,, 20 ,, 30 |
| ,, 27,000 ,, 64,000 ,, ,, ,, 30 ,, 40 |
etc.   etc.

Thus the finding of the circle on which to read the result presents no difficulty.

Logarithms.—The characteristic of a log. is the part to the left of the decimal point. It may be positive or negative.

The mantissa of a log. is the part to the right of the decimal point, and it is always positive. When multiplying or dividing for powers or roots, these points must be borne in mind.

The characteristic of the log., if positive, is one less than number of figures to left of decimal point ; if negative, one more than number of cyphers to right of decimal point.
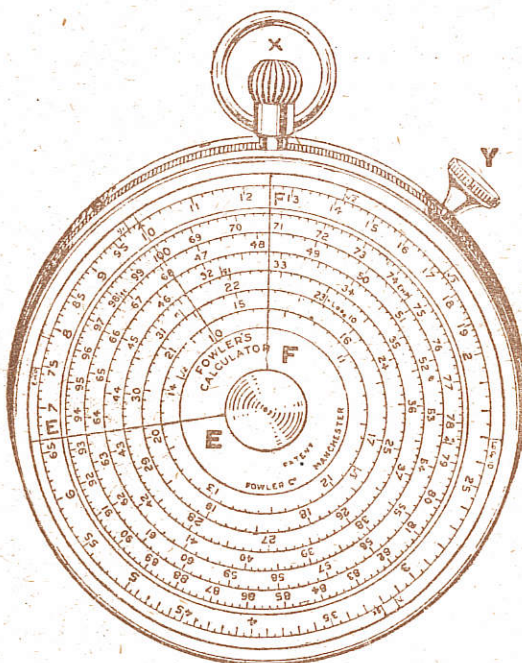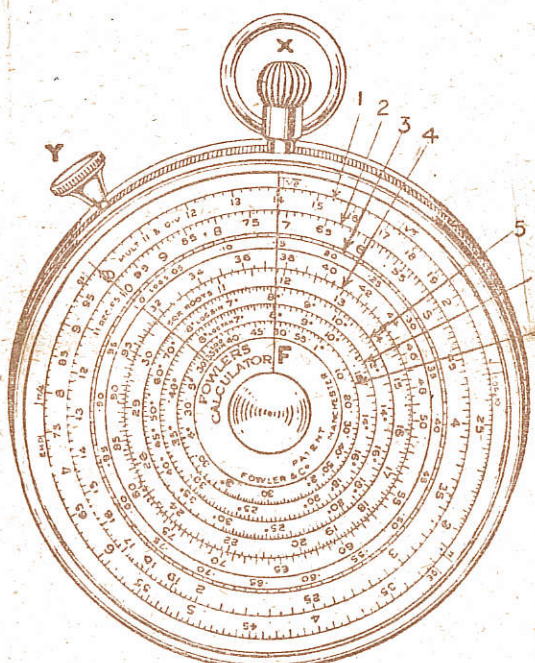

Fig. 1 Front Dial.

Fig. 2 Back Dial.

Example.—Find log. of 2675.

Set Cursor to 2675 on Scale 1. Read mantissa of log. viz. 427 on log. scale. The characteristic of the log. is 3 and the log. of 2675 = 3·427

Example.—Find log. of 50·75.

Set Cursor to 5075 on Scale 1. Read mantissa of log. viz. 7055 on the log. scale. The characteristic in this case is 1, and the log. of 50·75 = 1·7055.

The "Mantissa" part of the log. of a given number is independent of the position of the decimal point, but the characteristic changes if the decimal point changes position, e.g. :—

Log. 290 is 2·4624.   Log. ·29 is $\bar{1}$·4624.

Log. 29·0 is 1·4624.   Log. ·029 is $\bar{2}$·4624.

Log. 2·9 is 0·4624.   Log ·0029 is $\bar{3}$·4624.

Logarithms are very convenient for finding powers and roots of numbers. See Nth Powers and Roots.

Reciprocal Scale.—Scale 2 of the back dial affords a very convenient way of obtaining such values as :—

$$\frac{1}{a} \quad \frac{1}{a^2} \quad \frac{1}{\sqrt{a}} \quad \frac{1}{\sin a} \quad \frac{1}{\cos a} \quad \frac{1}{\tan a} \text{ etc.,}$$

by simply setting the Datum line over these respective values and reading the corresponding values on Scale No. 2. It will be noted further that with these two scales the decimal value of any fraction having 1 as a numerator, and any number as a denominator, can be instantly read. It matters not which scale is taken to represent the denominators, the other scale gives the decimal value. The product of any two opposed numbers on Scales No. 1 and No. 2 is, of course, equal to unity.

Percentages.—In the expression of percentages, attention should be paid to the basis on which it is measured. For instance, if A's salary is £75 and B's £50, it would be true to say that A's salary was 50 per cent greater than B's, taking B's salary as 100 per cent, and equally true to say that B's salary was 33⅓ per cent less than A's, taking A's salary as 100 per cent. The fact is only expressed in two different ways. There can be no misapprehension in any case if the quantity representing the 100 is made clear. Set the question as a problem in fractions, thus :—

Example.—In an examination, 27 scholars pass 1st class ; 35, 2nd class ; and 63, 3rd class. Express the various numbers as percentages.

The total number of scholars is 27 + 35 + 63 = 125.

If x, y, z are the three percentages, we have the following relationship :—

$$\frac{27}{125} = \frac{x}{100} \text{ and } x = \frac{100 \times 27}{125} = 21\cdot6 \text{ per cent.}$$

$$\frac{35}{125} = \frac{y}{100} \text{ and } y = \frac{100 \times 35}{125} = 28 \text{ per cent.}$$

$$\frac{63}{125} = \frac{z}{100} \text{ and } z = \frac{100 \times 63}{125} = 50\cdot4 \text{ per cent.}$$

For this class of question the instrument is very convenient. The fraction $\frac{100}{125}$ occurs in each, and by setting 1·0 under the Datum line and Cursor **E** to 125 (i.e., 12·5) on Scale 1, the several percentages can be read off at the Datum line **F** by rotating the dial until the several figures 27, 35, 63, on Scale 1, come under Cursor **E**.

Proportion.—Set the question in fractional form,

$$\frac{a}{b} = \frac{x}{c}$$

Where x is the unknown, and may be the numerator or denominator, as the operator finds convenient in setting the sum for solution. Then, by cross-multiplication,

$$a \times c = b \times x \text{ and } x = \frac{a \times c}{b}$$

Example 1.—15 men do a task in 28 days. In how many days should 21 men do it ?

Obviously more men will do the work in less time and we have the proportion :—

$$\frac{21}{15} = \frac{28}{x} \text{ and } x = \frac{28 \times 15}{21} = 20 \text{ days.}$$

Example 2.—If a task takes 40 days for 12 men, how many men will be required to do it in 24 days ?

Obviously more men will be required to do the task in less time, in the ratio of 40 to 24.

$$\frac{40}{24} = \frac{x}{12} \quad x = \frac{40 \times 12}{24} = 20 \text{ men.}$$

Natural, or Log. Sine ; Cosine ; Tangent, etc.—Use Datum line to read across from given angle on Scales Nos. 5, 6, 7, to Scale No. 1 for natural value of sine, tangent, etc., or to Scale No. 3 for log. values. Note following :—

Cos = sin of complement.
Cot = tan of complement.
Cos 60° = sin (90—60)° = sin 30°.
Cot 60° = tan (90—60)° = tan 30°.

All sines of angles on Scale No. 5 lie between 0·1 and 1·0 in value on Scale 1.

All sines of angles on Scale No. 7 (which are found on Scale 1 in a similar manner) lie between 0·01 and 0·1 in value on Scale 1.

Triangles ; Useful Notes.—If A, B, C is a triangle, the angles of which are A, B, C, and the sides opposite these angles are respectively a, b, c,

Then A + B + C = 180°.

$$\frac{a}{b} = \frac{\sin A}{\sin B} \qquad \frac{b}{c} = \frac{\sin B}{\sin C} \qquad \frac{c}{a} = \frac{\sin C}{\sin A}$$

$$a^2 = b^2 + c^2 - 2 \, bc \cos A.$$

N.B.—Cos A is itself minus, and the whole of the last factor becomes plus if A is greater than 90°.
If A is 90° the last factor disappears.
Sin (180—A) = sin A.
Cos (180—A) = —cos A.
Cotan A = tan (90—A).

$$\text{Area of a triangle} = \frac{a \, b \, \sin C}{2}$$

$$\tan \frac{A-B}{2} = \frac{a-b}{a+b} \cotan \frac{C}{2} \qquad \cotan \frac{C}{2} = \tan \frac{A+B}{2}$$

These last two formulæ are much used for solving triangles when two sides and included angle are known.

If in a triangle A, B, C, the angle C is a right angle (90°), and the sides opposite the angles are respectively a, b, c letters arranged clockwise

$$\text{Sin A} = \frac{a}{c} \qquad \sin B = \frac{b}{c} \qquad \cos A = \frac{b}{c} \qquad \cos B = \frac{a}{c}$$

$$\tan A = \frac{a}{b} \qquad \cotan A = \frac{b}{a} \qquad \text{secant A} = \frac{b}{c}$$

$$\text{cosecant A} = \frac{c}{a}$$

If C is not a right angle, the sine, cosine, etc., still have same values, but a and b are not now the sides of the actual triangle, but of an imaginary triangle with B C perpendicular to C A.

A circle is divided into 360 degrees. Each degree is divided into 60′ (minutes) ; each minute is divided into 60″ (seconds), but seconds are rarely considered.

For angles below about 6°, tangents can be read off on the sine scale for small angles (Scale 5).

The tangent is very slightly greater than the sine, but at 2° the error is only about one-sixteenth of one per cent, which is too small to be read on the calculator or any ordinary slide rule.

If the sine or tangent of an angle is less than 0·01, use scale Scale 7, but divide the angle by 10. Thus the angle whose tan is 0·00407 is one-tenth of 2° 20′; that is, 14 minutes. This is specially used in solving triangles with either two angles almost equal or with one very small angle.